

# Zassenhaus Group Theory Conference

Hosted by Adelphi University's Department of Mathematics and Computer Science

June 10-12, 2016

Adelphi University

Angello Alumni House

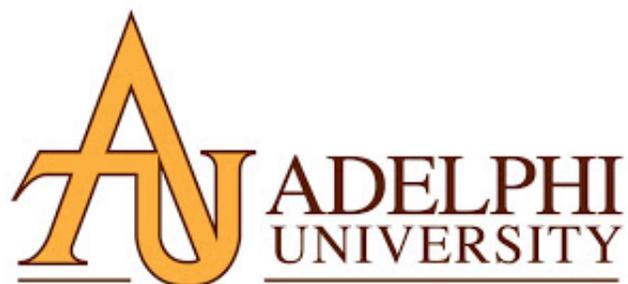
154 Cambridge Avenue, Garden City, NY 11530



Angello Alumni House



Hans Zassenhaus



## Friday, June 10, 2016; Afternoon Session

12:30-1:30 pm Registration

1:45-2:00 pm Sam L Grogg, Dean College of Arts & Sciences

Welcoming remarks

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2:00-2:20 pm Tuval Foguel (Adelphi University)

Idempotent Quasigroups and Exponent Two Loops

2:30-2:50 pm Elizabeth Wilcox (SUNY Oswego)

The Chermak-Delgado Lattice of a Finite Group: An update

3:00-3:20 pm Rachel K Skipper (Binghamton University)

Some branch groups with non-trivial rigid kernel

3:30-3:50 pm Bret Benesh (The College of Saint Benedict and Saint John's University)

Strategies for an Achievement and Avoidance Game on Symmetric and Alternating Groups

4:00-4:20 pm Arnold D. Feldman (Franklin & Marshall College)

Permutable Injectors

4:30-4:50 pm Zoran Sunic (Texas A & M University)

Language complexity of positive cones in free products

**8:00-9:00 am Coffee and Pastries**

**Saturday, June 11, 2016; Morning Session 1**

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9:00-9:20 am Viji Z. Thomas (The Indian Institute of Science Education and Research, Thiruvananthapuram)	Closure properties of the Non-abelian tensor product of groups and the Bogomolov multiplier
9:30-9:50 am Hung Ngoc Nguyen (The University of Akron)	An extension of Gluck's conjecture on the largest character degree
10:00-10:20 am Martha Kilpack (Brigham Young University)	For what finite lattices does the lattice of closure operators form a subgroup lattice?
10:30-10:50 am Arturo Magidin (University of Louisiana)	Algebraic closure operators on infinite subgroup lattices: preliminary report
11:00-11:20 am Christopher Cyr (University of Florida)	Nilpotent Subgroups and Semipermutability in the Symmetric Group
11:30-11:50 am Karl Lorensen (Penn State Altoona)	Torsion-free covers of virtually solvable minimax groups
12:00-12:20 pm Robert Griess (University of Michigan)	Integral forms in vertex operator algebras

**12:30-1:30 pm Lunch - Angello Alumni House**

## Saturday, June 11, 2016; Afternoon Session

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2:00-2:20 pm Patrizia Longobardi (Università di Salerno)	On a function on finite groups
2:30-2:50 pm Anthony Gaglione (U.S. Naval Academy)	Elementary and Universal Theories of Nonabelian CT and CSA Groups
3:00-3:20 pm Mandi A. Schaeffer Fry (Metropolitan State University of Denver)	On the Irreducible Restriction Problem for $SO(7, q)$
3:30-3:50 pm Delaram Kahrobaei (CUNY Graduate Center and New York City College of Technology)	Cryptography with right-angled Artin groups
4:00-4:20 pm Jonathan Gryak (CUNY Graduate Center)	On The Conjugacy Problem In Certain Metabelian Groups
4:30-4:50 pm Alexandre Turull (University of Florida)	Refinements of Dade's Projective Conjecture for $p$ -solvable groups

**Saturday, June 11, 2016; Conference Banquet - Angello Alumni House  
6:00 pm Cash Bar and Buffet Dinner**

**8:30-9:30 am Coffee and Pastries**

## **Sunday June 12, 2016; Morning Session**

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9:30-9:50 am Paul Baginski (Fairfield University)	Model Theoretic Advances for Groups With Bounded Chains of Centralizers
10:00-10:20 am Eric Swartz (College of William & Mary)	Covering symmetric groups with proper subgroups
10:30-10:50 am Gerhard Rosenberger (Universität Hamburg)	$i$ as a Quadratic Residue in the Gaussian Integers
11:00-11:20 am James Cossey (University of Akron)	The minimal number of characters in a block of a solvable group
11:30-11:50 am Luise-Charlotte Kappe (Binghamton University)	The nonabelian tensor product of cyclic groups of $p$ -power order: the compatibility question
12:00-12:20 pm Matt Visscher	Computing the nonabelian tensor product of cyclic groups of $p$ -power order

# Abstracts

Title: Idempotent Quasigroups and Exponent Two Loops

Presenter: Tuval Foguel

Affiliation: Adelphi University

Abstract: in this talk we prove that there is a one to one correspondence between finite idempotent quasigroup and finite loops of exponent 2. And we will use this correspondence to study some interesting loops of exponent two and some interesting idempotent quasigroup.

Title: The Chermak-Delgado Lattice of a Finite Group: An update

Presenter: Elizabeth Wilcox

Affiliation: SUNY Oswego

Abstract: Currently there are several avenues of research into the Chermak-Delgado lattice of a finite group, including work on groups with a trivial center (see McCulloch and Brewster), additional research into p-groups, efforts to build groups with a given Chermak-Delgado lattice, and extensions of the definition of the Chermak-Delgado lattice. This talk will explore some of the options for the latter task, discussing a few of the pitfalls encountered in attempting to generalize the Chermak-Delgado lattice concept.

Title: Some branch groups with non-trivial rigid kernel

Presenter: Rachel K Skipper

Affiliation: Binghamton University

Abstract: Since the construction of the Grigorchuk group in 1980, the study of branch groups has developed into a rich area of group theory. In this talk, we will discuss the congruence subgroup problem for branch groups which can be solved by computing the branch and rigid kernels. We will focus on groups for which the latter is non-trivial.

Title: Strategies for an Achievement and Avoidance Game on Symmetric and Alternating Groups

Presenter: Bret Benesh

Affiliation: The College of Saint Benedict and Saint John's University

Abstract: Let  $G$  a nontrivial finite group, and consider two games DNG and GEN with the following set-up: two players alternately select previously-unselected elements from  $G$ , and then determine the subgroup generated by all of the selected elements at the end of each turn. In DNG, the first player to generate  $G$  loses; in GEN, the first player to generate  $G$  wins. We discuss the strategies and nim-numbers for these games with alternating and symmetric groups. The alternating group case will require us to determine exactly when the maximal subgroups of even order cover an alternating group.

Title: Permutable Injectors

Presenter: Arnold D. Feldman

Affiliation: Franklin & Marshall College

Abstract: If  $\Sigma$  is a Hall system of a finite solvable group  $G$ , then a subgroup  $K$  of  $G$  is  $\Sigma$ -permutable if  $KH = HK$  for every subgroup  $H \in \Sigma$ . An injector is called permutable if it is  $\Sigma$ -permutable for some Hall system  $\Sigma$ . This talk will describe the significance of the permutability of injectors and an example of permutable injectors detailed in recent work with Rex Dark and Maria Dolores Perez-Ramos.

Title: Language complexity of positive cones in free products

Presenter: Zoran Sunic

Affiliation: Texas A & M University

Abstract: It is known that the positive cone of an order on a free product of groups cannot be finitely generated as a semigroup (Rivas, 2012). By using different methods, we extend this result to show that the positive cone in free products cannot be represented by a regular language. On the other hand, we exhibit countably many examples of orders on the free group of finite rank for which the positive cones are represented by context-free counter languages. Taken together, our result provide "sharp bound" on the language complexity of positive cones in free products. Parts of the talk are based on joint work with Susan Hermiller.

Title: Closure properties of the Non-abelian tensor product of groups and the Bogomolov multiplier

Presenter: Viji Z. Thomas

Affiliation: The Indian Institute of Science Education and Research, Thiruvananthapuram

Abstract: We will prove some new closure properties of the non-abelian tensor product of groups, recover a few old ones and pose some open questions. We will also prove the triviality of the Bogomolov multiplier for a large class of groups.

Title: An extension of Gluck's conjecture on the largest character degree

Presenter: Hung Ngoc Nguyen

Affiliation: The University of Akron

Abstract: An old conjecture of Gluck states that if  $G$  is a finite solvable group, then  $|G : F(G)| \leq b(G)^2$  where  $F(G)$  denotes the Fitting subgroup and  $b(G)$  the largest degree of an irreducible character of  $G$ . We prove the conjecture in the case when  $|F(G)|$  is coprime to 6, and then discuss an extension to arbitrary finite groups. This is a joint work with Cossey, Halasi, and Maroti.

Title: For what finite lattices does the lattice of closure operators form a subgroup lattice?

Presenter: Martha Kilpack

Affiliation: SUNY Oneonta

Abstract: If  $L$  is a lattice, the collection of all closure operators on  $L$  forms a lattice from a natural partial order. A standard example of a lattice is  $\text{Sub}(G)$ , the lattice of subgroups of a given group  $G$ . Continuing on our previous work, we determine all the finite lattices  $L$  for which the lattice of closure operators on  $L$  give a lattice that is isomorphic to  $\text{Sub}(G)$  for some group  $G$ .

Title: Algebraic closure operators on infinite subgroup lattices: preliminary report

Presenter: Arturo Magidin

Affiliation: University of Louisiana

Abstract: Let  $G$  be a group, and let  $L = \text{subgrps}(G)$  be the lattice of subgroups of  $G$ . A closure operator on  $L$  is algebraic if  $\phi(H)$  is the subgroup generated by all  $\phi(K)$ , where  $K$  are the finitely generated subgroups of  $H$ . The collection of all algebraic closure operators on  $L$  forms an algebraic lattice. Continuing on our previous work, we seek to determine for which groups  $G$  is this lattice,  $\text{aco}(\text{subgrps}(G))$ , isomorphic to the lattice of all subgroups of some group  $\mathcal{G}$ . We present several results for this case. We conjecture that for infinite groups  $G$ ,  $\text{aco}(\text{subgrps}(G))$  is isomorphic to a subgroup lattice if and only if  $G$  is isomorphic to a Prüfer  $p$ -group for some prime  $p$ .

Title: Nilpotent Subgroups and Semipermutability in the Symmetric Group

Presenter: Christopher Cyr

Affiliation: University of Florida

Abstract: We present a recent conjecture (2014) of Martin Isaacs concerning the normal closure of a nilpotent,  $S$ -semipermutable subgroup of a finite group  $G$ . We demonstrate that the conjecture is false in general by giving some counterexamples, but outline a proof that the conjecture does hold inside of the symmetric group  $S_n$ .

Title: Torsion-free covers of virtually solvable minimax groups

Presenter: Karl Lorensen

Penn State Altoona

Abstract: I discuss a new theorem, proved jointly with Peter Kropholler, stating that every finitely generated, virtually solvable minimax group can be realized as a quotient of a torsion-free, virtually solvable minimax group. One application of this result is that it solves a thirteen-year-old problem about random walks on the Cayley graphs of finitely generated, virtually solvable minimax groups.

We employ a novel approach to proving the theorem which involves embedding the minimax group densely in a locally compact group. This method also allows us to completely characterize the virtually solvable minimax groups that are homomorphic images of torsion-free, virtually solvable minimax groups.

Title: Integral forms in vertex operator algebras

Presenter: Robert Griess

Affiliation: University of Michigan

Abstract: An integral form in an algebra over a field of characteristic 0 is a free abelian group which is spanned by a basis and which is closed under the product.

Recently, Chongying Dong and I showed that there is a broad class of VOAs having an integral form (closed under the countable set of VOA compositions) which is invariant under the certain finite groups of automorphisms. This class includes the Moonshine VOA and integral forms invariant under the Monster finite simple group.

Ching Hung Lam and I studied a lattice type VOA for a root lattice of type ADE and defined generators of a Chevalley group on the VOA, using its standard integral form. When the form is tensored with a field, we get a vertex algebra (VA) and familiar Chevalley group over that field acting as automorphisms. Types BCFG are also treated. Thus, all Chevalley groups and Steinberg variations over finite fields (hence “most” finite simple groups) are essentially the full automorphism groups of a classical type VA over that finite field.

An old conjecture about Modular Moonshine (positive characteristic) of Borchers and Ryba was proved by Lam and me with a covering procedure and integral forms. We derive a new and theoretical proof that the sporadic simple group  $F_3$  of Thompson embeds in the Chevalley group  $E_8(3)$  over the field of 3 elements (this embedding was proved originally with computers in 1974).

Title: On a function on finite groups

Presenter: Patrizia Longobardi

Affiliation: Università di Salerno

Abstract: Let  $G$  be a periodic group. The problem of obtaining information about the structure of  $G$  by looking at the orders of its elements has been considered by many authors, from many different points of view.

In this talk we consider a finite group  $G$ , and we study the function on the element orders of  $G$  defined by

$$\psi(G) = \sum_{x \in G} o(x),$$

where  $o(x)$  denotes the order of the element  $x$ .

In 2009 H. Amiri, S.M. Jafarian Amiri and M. Isaacs proved that if  $G$  has order  $n$  and  $C_n$  denotes the cyclic group of order  $n$ , then

$$\psi(G) \leq \psi(C_n),$$

and

$$\psi(G) = \psi(C_n) \text{ if and only if } G \simeq C_n.$$

Other results have been obtained by H. Amiri, S.M. Jafarian Amiri, Y. Marefat, A. Iranmanesh, A. Tehranian,

I will discuss some new results concerning the structure of the group  $G$  assuming some inequalities involving  $\psi(G)$ , jointly obtained with Marcel Herzog and Mercede Maj.

Some other functions on the orders of the elements of a finite group  $G$  have been recently investigated by M. Garonzi and M. Patassini.

#### REFERENCES

- [1] H. Amiri, S.M. Jafarian Amiri and I.M. Isaacs, *Sums of element orders in finite groups*, Comm. Algebra **37** (2009), 2978-2980.
- [2] H. Amiri and S.M. Jafarian Amiri, *Sum of element orders on finite groups of the same order*, J. Algebra Appl. **10** (2011), 187-190.
- [3] S.M. Jafarian Amiri, *Characterization of  $A_5$  and  $PSL(2,7)$  by sum of elements orders*, Int. J. Group Theory **2** (2013), 35-39.
- [4] M. Garonzi and M. Patassini, *Inequalities detecting structural properties of a finite group*, arXiv:1503.00355v2 [math.GR] 26 december 2015.
- [5] M. Herzog, P. Longobardi and M. Maj, *On a function defined on the element orders of a finite group*, in preparation.
- [6] Y. Marefat, A. Iranmanesh and A. Tehranian, *On the sum of elements of finite simple groups*, J. Algebra Appl., **12** 7 (2013), 135-138.

Title: Elementary and Universal Theories of Nonabelian CT and CSA Groups

Presenter: Anthony Gaglione

Affiliation: U.S. Naval Academy

Abstract: Given a nonabelian commutative transitive (CT) group  $G$ , we show that the elementary theory of  $G$  is axiomatizable by the set  $H(G)$  of Horn sentences true in  $G$  together with CT. We also obtain results about the elementary theory of a group  $N$  free in the variety of all groups nilpotent of class at most  $c$ . Finally for a nonabelian CSA group  $G$ , we show that its universal theory is axiomatizable by the set  $Q(G)$  of quasi-identities true in  $G$  together with CT and the diagram of  $G$ . This last result generalizes a result of Myasnikov and Remeslennikov.

Title: On the Irreducible Restriction Problem for  $SO(7, q)$

Presenter: Mandi A. Schaeffer Fry

Affiliation: Metropolitan State University of Denver

Abstract: I will discuss the problem of restricting cross-characteristic representations of  $SO(7, q)$  to maximal subgroups, which is motivated by a theorem of G. Seitz and the Aschbacher-Scott program on classifying maximal subgroups of finite classical groups.

Title: Cryptography with right-angled Artin groups

Presenter: Delaram Kahrobaei

Affiliation: CUNY Graduate Center and New York City College of Technology

Abstract: In this talk we propose right-angled Artin groups as platform for a secret sharing scheme based on the efficiency (linear time) of the word problem. We define two new problems: subgroup isomorphism problem for Artin subgroups and group homomorphism problem in right-angled Artin groups. We show that the group homomorphism and graph homomorphism problems are equivalent which is known to be NP-complete. In the case of sub-group isomorphism problem, we bring some results due to Bridson who shows there are right-angled Artin groups in which this problem is unsolvable. We propose two authentication schemes based on subgroup isomorphism problem and group homomorphism problem in right-angled Artin groups. Note that our schemes are similar to what Grigoriev and Shpilrain proposed for graphs.

This is a joint work with Ramón Flores from University of Seville (Spain).

Title: On The Conjugacy Problem In Certain Metabelian Groups

Presenter: Jonathan Gryak

Affiliation: CUNY Graduate Center

Abstract: We analyze the computational complexity of the conjugacy search problem in a certain family of metabelian groups. We prove that in general the time complexity of the conjugacy search problem for these groups is at most exponential. For a subfamily of groups we prove that the conjugacy search problem is polynomial. We also show that for some of these groups the conjugacy search problem reduces to the discrete logarithm problem. We provide some experimental evidence which illustrates our results probabilistically.

This is a joint work with Delaram Kahrobaei and Conchita Martinez-Perez.

Title: Refinements of Dade's Projective Conjecture for  $p$ -solvable groups

Presenter: Alexandre Turull

Affiliation: University of Florida

Abstract: A number of well known conjectures in representation theory of finite groups relate the properties of the representations of a finite group with the properties of the representations of its local subgroups. The most well known of these conjectures include the McKay Conjecture and the Alperin Weight Conjecture. In 1994, Dade proposed his Projective Conjecture which, if true, would imply both the McKay Conjecture and Alperin's Weight Conjecture. All these conjectures remain open in general. In 2000, G. R. Robinson proved Dade's Projective Conjecture for  $p$ -solvable groups. Refinements of Dade's Projective Conjecture have been proposed by Uno and Boltje, and these have been studied for  $p$ -solvable groups by Glesser. Turull proposed a different refinement of the McKay Conjecture. In this talk, we adapt Turull's Conjecture to the point of view of Dade's Projective Conjecture, and to Boltje's Conjecture. We then discuss the proofs of the resulting conjectures (with all the above mentioned refinements) for all  $p$ -solvable finite groups. As a consequence, we obtain Uno's refinement of the Dade Projective Conjecture for all  $p$ -solvable finite groups. The proof develops a Clifford theory for alternating sums of numbers of characters which respects the rationality of the characters.

Title: Model Theoretic Advances for Groups With Bounded Chains of Centralizers

Presenter: Paul Baginski

Affiliation: Fairfield University

Abstract: A group  $G$  has bounded chains of centralizers ( $G$  is  $\mathfrak{M}_C$ ) if every chain of centralizers  $C_G(A_1) \leq C_G(A_2) \leq \dots$  is finite. While this class of groups is interesting in its own right, within the field of logic known as model theory,  $\mathfrak{M}_C$  groups have been examined because they strictly contain the class of stable groups. Stable groups, which have a rich literature in model theory, robustly extend ideas from algebraic groups, such as dimension and independence, to a wider setting, including free groups. Stable groups gain much of their strength through a chain condition known as the Baldwin-Saxl chain condition, which implies the  $\mathfrak{M}_C$  property as a special case.

Several basic, but key, properties of stable groups have been observed by Wagner [4] and others to follow purely from the  $\mathfrak{M}_C$  condition (this builds upon the work of Bludov [2], Khukhro [3], and others). These properties were, as one would expect, purely group-theoretic. From the perspective of logic, the class of  $\mathfrak{M}_C$  groups should be unruly, since the  $\mathfrak{M}_C$  cannot be captured using first-order axioms (unless one insists on a fixed finite centralizer dimension). Yet the speaker and Tuna Altinel [1] have recently uncovered that  $\mathfrak{M}_C$  groups possess a logical property of stable groups as well, namely the abundance of definable nilpotent subgroups. We shall present this result and describe the current investigations for finding an analogue for solvable subgroups.

#### REFERENCES

- [1] T. Altinel and P. Baginski, *Definable envelopes of nilpotent subgroups of groups with chain conditions on centralizers*, to appear in Proc. Amer. Math Soc.
- [2] V.V. Bludov, *On locally nilpotent groups with the minimality condition for centralizers*, Algebra Logika **37** (1998), 270–278.
- [3] E.I. Khukhro, *On solubility of groups with bounded centralizer chains*, Glasg. Math. J. **51** (2009), 49–54.
- [4] F.O. Wagner, **Stable Groups**, London Mathematical Society Lecture Note Series, 240. Cambridge University Press, 1997.

Title: Covering symmetric groups with proper subgroups

Presenter: Eric Swartz

Affiliation: College of William & Mary

Abstract: If a group  $G$  is the union of proper subgroups  $H_1, \dots, H_k$ , we say that the collection  $\{H_1, \dots, H_k\}$  is a cover of  $G$ , and the size of a minimal cover (supposing one exists) is the covering number of  $G$ , denoted by  $\sigma(G)$ . Maróti showed that  $\sigma(S_n) = 2^{n-1}$  for  $n$  odd and sufficiently large, and he also gave asymptotic bounds for  $n$  even. In this talk, we will discuss these previous results and give the exact value of  $\sigma(S_n)$  when  $n$  is divisible by 6.

Title:  $i$  as a Quadratic Residue in the Gaussian Integers

Presenter: Gerhard Rosenberger

Affiliation: Universität Hamburg

Abstract: We use the structure of the Generalized Picard Group  $\text{PGL}(2, \mathbb{Z}[i])$  to provide a characterization of Gaussian integers for which  $i$  is a quadratic residue. This is a two-square theorem analogous to Fermat's two-square theorem.

Title: The minimal number of characters in a block of a solvable group

Presenter: James Cossey

Affiliation: University of Akron

Abstract: Let  $B$  be a block of the solvable group  $G$  with defect group  $D$ . It is easy to show, using Brauer's main theorems, that  $k(B) \leq k(G, D)$ , where  $k(B)$  denotes the number of ordinary characters in  $B$  and  $k(G, D)$  denotes the number of conjugacy classes of  $G$  that intersect  $D$ . We show that if equality occurs, then  $B$  is nilpotent and in fact  $k(B) = k(D) = k(G, D)$ . This is related to a question of Isaacs and Navarro regarding the minimal number of characters over a  $G$ -invariant character of a normal subgroup, which we also answer for solvable groups. Along the way, we will develop the notion of a fully ramified coprime action, which is useful in other applications as well.

Title: The nonabelian tensor product of cyclic groups of  $p$ -power order: the compatibility question

Presenter: Luise-Charlotte Kappe

Affiliation: Binghamton University

Abstract: The non-abelian tensor product of a pair of groups was introduced by R. Brown and J.-L. Loday. It arises in the applications in homotopy theory of a generalized Van Kampen theorem.

Let  $G$  and  $H$  be groups which act on each other via automorphisms and which act on themselves via conjugation. The actions are said to be compatible if  ${}^s h g' = {}^g ({}^h (g^{-1} g'))$  and  ${}^h s h' = {}^h (s ({}^{h^{-1}} h'))$  for all  $g, g' \in G$  and  $h, h' \in H$ . The nonabelian tensor product  $G \otimes H$  is defined provided  $G$  and  $H$  act compatibly. In such a case  $G \otimes H$  is the group generated by the symbols  $g \otimes h$  with relations  $g g' \otimes h = ({}^g g' \otimes {}^g h)(g \otimes h)$  and  $g' = (g \otimes h)({}^h g \otimes {}^h h')$ .

If  $G = H$ , we call  $G \otimes H$  the tensor square of  $G$ . Here the action is conjugation which is always compatible. Good progress has been made in determining the nonabelian tensor square for large classes of groups. However in the case of nonabelian tensor products the enigma of compatible actions has prevented such progress. Only in a few cases the nonabelian tensor product of two groups with nontrivial compatible actions has been determined.

Here we establish a necessary and sufficient condition that a pair of actions on two cyclic groups is a compatible pair. With its help we classify all compatible actions in the case of cyclic  $p$ -groups, paving the way for the determination of the nonabelian tensor product of cyclic groups of  $p$ -power order.

This is joint work with M.P. Visscher and M.S. Mohamad.

Title: Computing the nonabelian tensor product of cyclic groups of  $p$ -power order

Presenter: Matt Visscher

Abstract: Let  $G$  and  $H$  be groups which act on each other via automorphisms and which act on themselves via conjugation. The actions are said to be compatible if  ${}^s h g' = {}^g ({}^h (g^{-1} g'))$  and  ${}^h s h' = {}^h (s ({}^{h^{-1}} h'))$  for all  $g, g' \in G$  and  $h, h' \in H$ . The nonabelian tensor product  $G \otimes H$  is defined provided  $G$  and  $H$  act compatibly. In such a case  $G \otimes H$  is the group generated by the symbols  $g \otimes h$  with relations  $g g' \otimes h = ({}^g g' \otimes {}^g h)(g \otimes h)$  and  $g' = (g \otimes h)({}^h g \otimes {}^h h')$ .

The nonabelian tensor product was introduced by R. Brown and J.-L. Loday in connection with applications in homotopy theory of a generalized Van Kampen theorem.

Good progress has been made in the computation of large classes of groups in the case of the nonabelian tensor square  $G \otimes G$ , where the actions are conjugation which are always compatible. A few nonabelian tensor products of small cyclic groups were determined. They all turned out to be cyclic. However in 1989 Gilbert and Higgins established that the nonabelian tensor product of two infinite cyclic groups, where the mutual actions are inversion, is isomorphic to the free abelian group of rank 2, contradicting an earlier conjecture that the nonabelian tensor product of two cyclic groups is always cyclic.

Here we show that the nonabelian tensor product of the cyclic groups is an abelian group of at most rank 2. The nonabelian tensor product of two cyclic  $p$ -groups of  $p$ -power order is always cyclic of  $p$ -power order with the exception of some 2-groups with certain actions of order 2, where the non abelian tensor product is an abelian 2-group of rank 2.

This is joint work with L.-C. Kappe and M.S. Mohamad.